Reg. No.: | | | | | | | | |

G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.

PG DEGREE END SEMESTER EXAMINATIONS - NOVEMBER 2024.

(For those admitted in June 2023 and later)

PROGRAMME AND BRANCH: M.Sc., MATHEMATICS

SEM	CATEGORY	COMPONENT	COURSE CODE	COURSE TITLE
III	PART - III	CORE-8	P23MA308	DIFFERENTIAL GEOMETRY

Date : 08.11.2024 / FN **Time : 3 hours** **Maximum: 75 Marks**

Course Outcome	Bloom's K-level	Q. No.	**SECTION – A (10 X 1 = 10 Marks)** Answer ALL Questions.
CO1	K1	1.	Identify the equation $[\boldsymbol{R}-\boldsymbol{r}(0), \boldsymbol{r}'(0), \boldsymbol{r}(s)-\boldsymbol{r}(0)]$ from the following: a) osculating plane b) tangent plane c) normal plane d) secant plane
CO1	K2	2.	The _____ at a point P is the line of intersection of the normal plane and the osculating plane at P. a) normal plane b) principal normal c) binormal d) tangent
CO2	K1	3.	Which of the following is the radius of spherical curvature? a) $R=(\rho^2+\sigma^2)^{\frac{1}{2}}$ b) $R=(\rho^2+\sigma^2\rho')^{\frac{1}{2}}$ c) $R=(\rho^2+\sigma^2\rho'^2)^{\frac{1}{2}}$ d) $R=(\rho'^2+\sigma^2\rho^2)^{\frac{1}{2}}$
CO2	K2	4.	____ of C is a curve which lies on the tangent surface of C and intersects the generators orthogonally. a) tangent b) normal c) involute d) evolute
CO3	K1	5.	For the paraboloid, $r_1=(1,0,2u)$, $r_2=(0,1,-2v)$, then $F=$ ______. a) $1+4u^2$ b) $1+4v^2$ c) $-4uv$ d) $1-4uv$
CO3	K2	6.	The two families are orthogonal if and only if_____. a) $ER-2FQ-GP=0$ b) $ER+2FQ+GP=0$ c) $ER+2FQ-GP=0$ d) $ER-2FQ+GP=0$
CO4	K1	7.	At every point the principal normal of a geodesic is ______ to the surface. a) tangent b) normal c) parallel d) intersect
CO4	K2	8.	For any curve on a surface, the curvature vector at a point P is. a) $\boldsymbol{r}'=\kappa\boldsymbol{n}$ b) $\boldsymbol{r}'=\tau\boldsymbol{n}$ c) $\boldsymbol{r}''=\kappa\boldsymbol{n}$ d) $\boldsymbol{r}''=\tau\boldsymbol{n}$
CO5	K1	9.	The second fundamental form is_____. a) $Ldu+2Mdudv+Ndv$ b) $Ldu-2Mdudv+Ndv$ c) $Ldu^2+2Mdudv+Ndv^2$ d) $Ldu^2-2Mdudv+Ndv^2$
CO5	K2	10.	Gaussian curvature κ is defined as a) $K=\frac{LN-M}{EG-F}$ b) $K=\frac{LN-M^2}{EG-F^2}$ c) $K=\frac{LN+M}{EG+F}$ d) $K=\frac{LN+M^2}{EG+F^2}$

Course Outcome	Bloom's K-level	Q. No.	**SECTION – B (5 X 5 = 25 Marks)** Answer ALL Questions choosing either (a) or (b)
CO1	K2	11a.	Show that $[\dot{r},\ddot{r},\dddot{r}]=0$ is a necessary and sufficient condition that the curve be plane. **(OR)**
CO1	K2	11b.	Show that the necessary and sufficient condition for a curve to be a straight line is $\kappa=0$ for all points of the curve.

CO2	K2	12a.	Show that the characteristic property of helices is that the ratio of the curvature to the torsion is constant at all points.
			(OR)
CO2	K2	12b.	Trace the equation which represent an infinite system of evolutes of the given curve.
CO3	K3	13a.	Calculate the first fundamental coefficients of the anchor ring.
			(OR)
CO3	K3	13b.	On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes z =constant.
CO4	K3	14a.	Write down the proof of the curves of the family $\frac{v^3}{u^2}$ =constant are geodesics on the surface with the metric $v^2du^2 - 2uvdudv + 2u^2dv^2$ where $u > 0, v > 0$.
			(OR)
CO4	K3	14b.	Write down the proof of that if the orthogonal trajectories of the curves $v =$ constant are geodesics, then $\frac{H^2}{E}$ is independent of u.
CO5	K4	15a.	Prove that if there is a surface of minimum area passing through the closed space curve, it is a minimal surface that is a surface of zero mean curvature
			(OR)
CO5	K4	15b.	State and prove Rodrigue's formula.

Course Outcome	**Bloom's K-level**	**Q. No**	**<u>SECTION – C</u> (5 X 8 = 40 Marks)** **Answer <u>ALL</u> Questions choosing either (a) or (b)**
CO1	K4	16a.	Examine the curvature and torsion of the cubic curve given by $\boldsymbol{r} = (\boldsymbol{u}, \boldsymbol{u}^2, \boldsymbol{u}^3)$.
			(OR)
CO1	K4	16b.	Inspect the curvature and torsion of the curve of intersection of two quadratic surfaces $ax^2 + by^2 + cz^2 = 1, a'x^2 + b'y^2 + c'z^2 = 1$.
CO2	K5	17a.	Prove that if the radius of spherical curvature is constant, the curve either lies on a sphere or has constant curvature.
			(OR)
CO2	K5	17b.	Prove that if C is a spherical helix, then the plane C_1 perpendicular to its axis is an arc of an epicycloid.
CO3	K5	18a.	Deduct the surface of revolution which is isometric with the region of the right helicoid.
			(OR)
CO3	K5	18b.	If (l', m') are the direction coefficient of line which makes an angle $\frac{\pi}{2}$ with the line whose direction coefficients are (l, m), then prove that $l' = -\frac{1}{H}(Fl + Gm)$ and $m' = \frac{1}{H}(El + Fm)$
CO4	K5	19a.	Prove that on the general surface a necessary and sufficient condition that the curve $v = c$ be a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$ when $v = c$ for all values of v.
			(OR)
CO4	K5	19b.	State and prove Gauss-Bonnet theorem.
CO5	K6	20a.	Prove that a necessary and sufficient condition for a surface to be developable is that its Gaussian curvature shall be zero.
			(OR)
CO5	K6	20b.	Prove that a necessary and sufficient condition that a curve on a surface be a line of curvature is that the surface normal along the curve form a developable.